Sub Array Effect on MUSIC for Partial Adaptive Array Radar

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***Abstract* - Eigen Sub space methods like Multiple Signal Classification (MUSIC) has drawn much attention in the field of Direction of Arrival estimation with antenna arrays because of super resolution properties. In order to reduce the complexity and the computational load, designers are choosing the partial adaptive arrays i.e. total antenna array is divided in to sub array and sub array outputs are processed instead of element outputs. In this paper we discuss the limitation of the MUSIC algorithm for linear sub arrays with the simulation results.**

***Keywords* - MUSIC, Super Resolution.**

I. INTRODUCTION

Angular super resolution has become one of the important research topic. Super resolution in angular domain provides the solution of some problems but needs much computational power. Super resolution algorithms needs the knowledge of the array structure for efficient results and array calibration is needed. Most of the super resolution algorithms give efficient results at element level but it needs knowledge about the antenna element positions and requires high computational power. In order to reduce the complexity and computational load designers combines few elements and provides the sub array output.

Spectrum estimation using the sub-array outputs, need not have the knowledge of the complete array, only gains and the phase centres of the sub arrays are enough to compute the array manifold vector and subsequent spectrum[2]. The above method can provide the super resolution only around the look-direction, named as Spotlight MUSIC by U. Nickel [1].

In this paper the MUSIC spectrum estimation for linear array at sub array level with regular sub arrays are presented. Due to sampling the effect of grating lobe on the main lobe for MUSIC has been discussed with simulations. The grating lobe can be minimized by using the irregular sub array shapes.

The structure of the paper as follows: explanation of the MUSIC spectrum estimation by using sub arrays, Antenna element position and sub array configurations for simulations and lastly discussions on the simulation results followed by conclusion.

II. MUSIC SPECTRUM AT SUB ARRAY LEVEL

One dimensional linear array has been considered, which can steer the beam by using the phase changes. The inter element distance is considered as half wavelength. Assume that there are N narrow-band sources impinging on array from far-field direction of (θ_n) where the bore-sight of the array is (θ_0).The output of the l^{th} *(l = 1, 2.....L)* element, here we considered first element as reference is

$$x_l(t) = g_l \sum_{n=1}^{N} s_n(t)\, exp\left\{-j\frac{2*\pi}{\lambda}[(x_l - x_1)(\alpha_n - \alpha_0)]\right\} + n_l(t) \quad (1)$$

Where

$$\alpha_n = sin\theta_n$$

$$\alpha_0 = sin\theta_0$$

g_l is the gain of the element. Each element is assumed as Omni-directional elements and without tapering. The antenna is divided in to P number of non-overlapped sub arrays. $s_n(t)$ is the nth incident signal, $n_l(t)$ is the white Gaussian noise in the lth element.

Now we construct a Transformation matrix (T-matrix) from element level to sub array level. It has a dimension of *P X L*, where P is the number of sub arrays. Matrix will have zeros and ones. Ones are present in a particular row (sub array) for a given column (element).
From (1) we can obtain sub array output vector as

$$X(t) = Tx(t) \quad (2)$$

Where

$$X(t) = [X_1(t)\ , X_2(t), \dots\dots\dots . . X_P(t)\ ] \quad (3)$$

The sub array phase centers are γ_x and gain of the p^{th} sub array are given by (4), (5) and (6) respectively.

$$\gamma_x = \frac{\sum_{l\in U_p} g_l\, x_l}{\sum_{l\in U_p} g_l} \quad (4)$$

$$G_p = \sum_{l\in U_p} g_l \quad (5)$$

Since tapering is not applied and each element is a Omni-directional element, gain will be unity, then the above equations (4),(5) can be replaced with (6) and (7)

$$\gamma_x = \frac{\sum_{l\in U_p} x_l}{Q} \quad (6)$$

$$G_p = Q \qquad (7)$$

Where, Q is number of elements in the sub array.

For a given angle θ, the array manifold vector is

$$A(\theta) = [a_1(\theta), a_2(\theta), a_3(\theta) \ldots\ldots\ldots a_P(\theta)] \qquad (8)$$

Where

$$a_p(\theta) = G_p \exp\left\{-j\frac{2*\pi}{\lambda}\left[\left(\gamma_{x_p} - \gamma_{x_1}\right)(\sin\theta - \alpha_0)\right]\right\} \qquad (9)$$

The estimated power density for any given angle θ is as follows

$$S_{sub}(\theta) = \frac{A(\theta)^H A(\theta)}{A(\theta)^H E_n E_n^H A(\theta)} \qquad (10)$$

Where E_n is a noise-eigen vector matrix which is complementary of signal sub-space. E_n is extracted from the covariance matrix given in (11)

$$R = E[\boldsymbol{X}(\boldsymbol{t})\boldsymbol{X}(\boldsymbol{t})^{\boldsymbol{H}}] \qquad (11)$$

The dimension of R is P X P.

III. Antenna Element and Sub Array Configuration

In this paper a uniform linear array(ULA) of 64 elements with a half-wavelength inter element spacing is considered. The antenna is divided into 8 regular sub arrays with each sub array of 8 elements. The configuration is explained in Table1. Since all sub arrays has the same number of elements, gain of all sub arrays are equal as shown in “Fig. 1”. Here we have not applied any tapering, if we apply tapering, the gain of each sub array will be different because it will depend on the amplitude weights of each element. The phase centers of the sub arrays are calculated using equation (6) and shown in “Fig. 2”, for irregular sub arrays with the same number of elements, the antenna is divided into 8 sub arrays with the configuration as shown in Table2. The sub array gains are not equal because the number of elements in each sub array is not equal and the pattern of each sub array will be different because of the shape of the sub array, as shown in “Fig. 3”. In this array the half-symmetry is maintained so only four sub array patterns are shown and remaining four sub arrays are duplicate of these patterns, The sub array phase centers are randomized and can be in “Fig. 4”.

Table1.Subarray and element configuration for regular subarrays

Element Index	1-8	9-16	17-24	25-32	33-40	41-48	49-56	57-64
Sub array Index	1	2	3	4	5	6	7	8

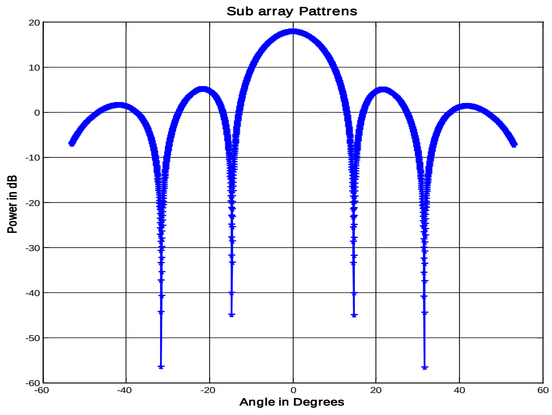

Fig. 1. *Regular Subarray pattrens*

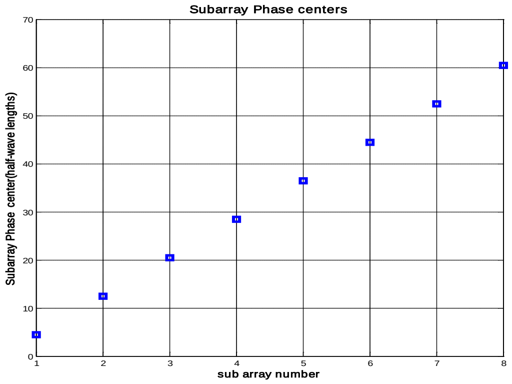

Fig. 2. *Regular Subarray phase centers*

Table2.Subarray and element configuration for ir- regular subarrays

Element Index	1-10	11-19	20-26	27-32	33-38	39-45	46-54	55-64
Sub array Index	1	2	3	4	5	6	7	8

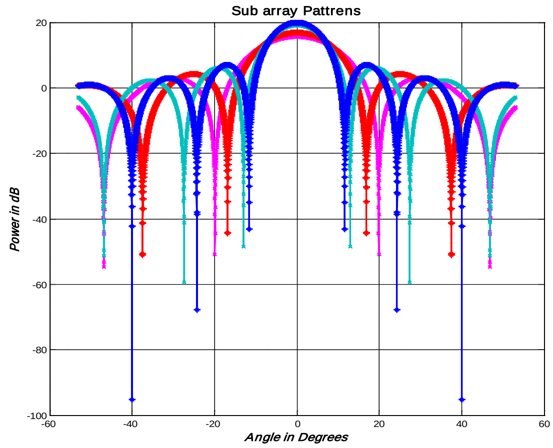

Fig. 3. *Irregular Subarray pattrens*

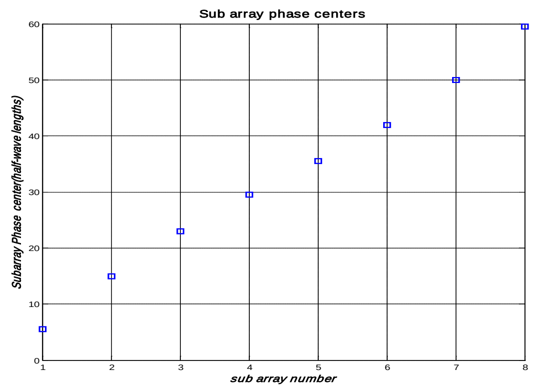

Fig. 4. *Iregular Subarray phase centers*

IV. Simulation Results

Here we assume two sources are impinging on the array with a SNR of 10 dB at element level from $+2^0$ and -2^0. Bore-sight of the array is 0^0. The total number of i.i.d samples used for covariance matrix estimation is 300. The above two antenna configurations are used for MUSIC spectrum estimation.

The direction of the sources chosen within the beam width of the sub array for accurate estimation[2]. The MUSIC algorithm has confined its search around the maximum beam width of the it's sub arrays as -10^0 to $+10^0$. The estimation of the direction for both the sources are acceptable as shown in “Fig. 5” by using regular sub arrays and “Fig. 6” by using irregular sub arrays.

If the direction of the first source is at -15^0 which is out of beam width for the regular and irregular sub arrays. As per MUSIC algorithm where it is searching only between -10^0 to $+10^0$ there should not be any other peak other than at $+2^0$, but there is a peak at -1^0 because of the grating lobe by using the regular sub arrays as shown in "Fig 7" and same has been eliminated by using the irregular sub arrays as shown in "Fig 8".

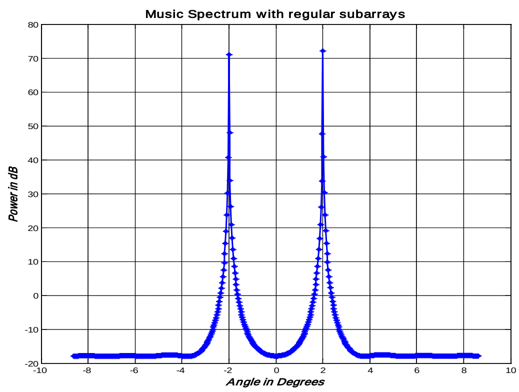

Fig. 5. *MUSIC spectrum for regular subarrays for sources at $+2^0$ and -2^0*

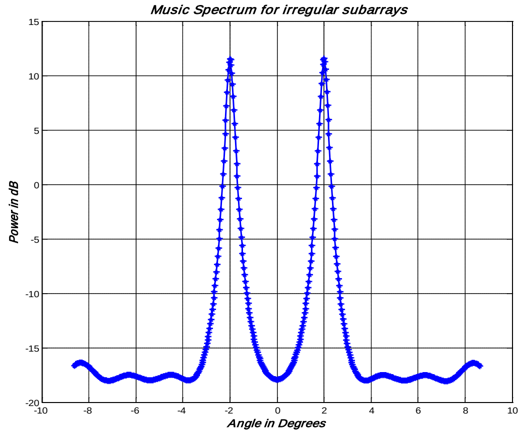

Fig. 6. *MUSIC spectrum for regular subarrays for sources at $+2^0$ ans -2^0*

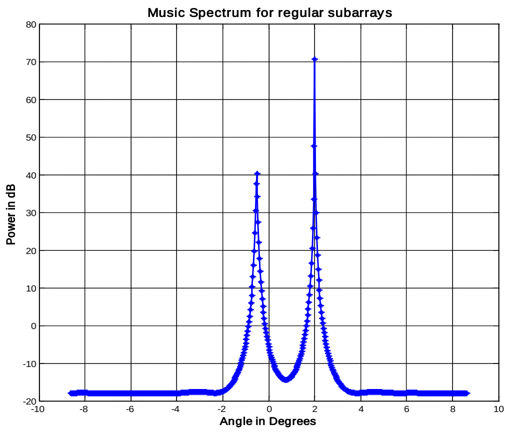

Fig. 7. *MUSIC spectrum for regular subarrays for sources at $+2^0$ ans -15^0*

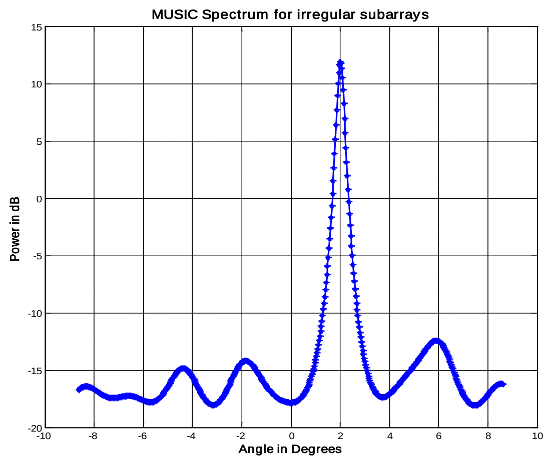

Fig. 8. *MUSIC spectrum for irregular sub arrays for sources at $+2^0$ ans -15^0*

V. Conclusion

We have discussed the limitations of MUSIC algorithm for regular sub arrays with a ULA. Depending on the phase centre of the sub arrays regular sub arrays will have grating lobes. If there is a source around the grating lobe,MUSIC will have the ambiguity in the DOA estimation. By using irregular sub-arrays the phase centres can be irregularzed and grating lobes can be avoided to some extent. So MUSIC is suitable only for irregular sub arrays and its performance depends on the sub array configuration also.

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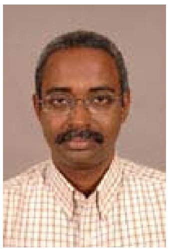

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